A Study on Features of Stone Skipping

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How to get more bounces?
Abstract

Stone skipping, as a leisure entertainment, is loved by many people. There are still many enthusiasts who want to challenge the world record of stone skipping and study on the features of it. However, few paper concentrates on both the theory and the experiment and makes systematic study. In this paper, the physics model is established to simulate the stone skipping process. The self-made equipments and self-written programs are used to finish our experiment. We study the influence of shape, contact surface, mass, velocity, angular velocity of stone on the number of bounces, the change of the slant angle and the deflected trajectory. The differences between the experiment results and the theoretical results are corrected by considering the load of water entry and some other factors. In this way we get the ultimate theoretical results which coincide with the experiment results and the world record. In the process of our study, some unexpected phenomena are explained such as the flight behavior of stone which can adjust automatically before falling into the water and the distance of bounces sometimes short and sometimes long in the world record video. They are never mentioned in other papers.
Statement of Originality

The research process and results of our group are conducted and derived under the guidance of the instructors. Except the referenced contents and the acknowledged source, this paper does not include any published findings by any group or any researcher.

Signature: Huang Yixuan, Jiang Weibo, Shen Ming
Date: Sept. 15th, 2016
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A Study on Features of Stone Skipping

1 Introduction

1.1 Background

Stone skipping is one of the traditional pastimes which history can be traced back to the Stone Age. This folk game was spread widely in south region of ancient China where rich in water resources. The stone skipping game is always organize in leisure time in Minorities, the Uygur, Tajik and Tatar in Guangdong Province and Xinjiang Uygur Autonomous Region.

Stone skipping is easy to implement and interesting which can be carried out by crock or flat stones. This competition is divided into single, double and multi person groups. Today many people still like this game because of its simplicity and widespread feature. The rules of game are quite simple: comparing the jumping times and distances of stone on the water. In August of this year, a 21-year-old Japanese, Gang YouShi, played an amazing record with 91 times of hydroplaning, broke Guinness Book of World Records created by Kurt Steiner in 2014. The stone threw by Kurt jumped 88 times of hydroplaning.

People skip the stone under the same conditions, some individuals always play a few hydroplaning, but some others can play dozens of hydroplaning. The relationship between the techniques of stone skipping and the principles of physics caused us great attraction. We will use specific quantitative analysis to deduce the kinematics and dynamics equations of stone skipping and discover the relationship of various parameters in hydroplaning movement. The experimental results will be used to compare with the mathematical models, and then provide
the guidance to revise the model.

1.2 Analysis

When we just started studying, we made some qualitative analysis inspired by the experience

Firstly, compared with curved stone, the flat one has a smaller mass, and is subjected to larger force. It is easier for the stone to bounce.

Secondly, it is important to have larger angular velocity because of the gyroscopic effect. The flying attitude of the stone is more stable when it has larger angular velocity.

Thirdly, in order to avoid sinking into the water, the velocity of the stone should be as fast as possible.

Finally, there might be a better performance when you choice a circular stone which mass distribution uniform.

And we also did some quantitative analysis.

After consulting literature material, we had known that the phenomenon of stone skipping is a kind of ‘Water sliding’, such as Dolphin-like movement. It will happen when the following conditions are met

\[ Fr = \frac{U^2}{gL} > 5 \]

When \( Fr \) is the Froude number, \( U \) is velocity, \( L \) is Characteristic length. In our experiment, \( L \approx 0.04 \text{m} \)

So

\[ U > 0.283 \text{m/s} \]
Therefore, we should make sure our launch device should meet this condition in our experiment.

In the following paper, we will try to design an innovation experiment and establish a simplified physical model which simulated the process of stone skipping.

2 The Description of Basic Physics

2.1 Basic Symbol Explanation

$\alpha$: the slant angle of stone ($^\circ$);

$\beta$: the angle between the velocity and the horizontal plane ($^\circ$);

$v_0$: initial velocity (m/s);

$m$: the mass of stone (m);

$C_r$: the coefficient of liquid pressure drag along the tangential direction;

$C_n$: the coefficient of liquid pressure drag along the normal direction;

$d$: the diameter of a circular stone or the length of a rectangular stone (m);

$\rho$: the density of water (kg/m$^3$);

$A$: the submerged area of stone (m$^2$);

$E_k$: kinetic energy (J);

$E_p$: potential (J).

2.2 The Definition of Kinematics

$e_x$: the unit vector which is tangential to the stone;

$e_z$: the unit vector which is parallel to the stone;

$x$: the direction of displacement to $x$ (m);
$z$: the direction of displacement to $z$ ($m$);

$v_{ox}$: the direction of initial velocity to $x$ ($m/s$);

$v_{oz}$: the direction of initial velocity to $z$ ($m/s$);

$a_x$: the acceleration direction of $x$ ($m/s^2$);

$a_z$: the acceleration direction of $z$ ($m/s^2$);

3 The Preliminary Construction of Models

3.1 Model Assumptions

Due to the stone skipping process involving the complex hydromechanics and aerodynamics, if all of these factors are considered, the mathematical model will become extremely complicated. Meanwhile, some tiny practical factors should be neglected. Therefore, some assumptions are employed as follows:

(1) The water is motionless or flow sufficiently slowly.

(2) There is no or sufficiently small wind.

(3) The water is clear with no suspended substance.

(4) The stone is firm enough and is not to break up at any time.

(5) The water is deep enough.

(6) The stone does not take out spray after it bounces.

3.2 Mathematical Models

When the object and liquid occurring relative motion, the object will be affected by the gravity, the pressure and the resistance caused by liquid, such as the viscosity resistance and the pressure drag. Compare with the other two kinds of resistance, the pressure drag play the
most important part.

Initially, the gravity force acts on the particle is greater than the liquid forces, and the particle will accelerate when moving to the underneath. After a short time, the particles suffers a larger pressure drag. As the external forces of particles are zero, it will be in uniform descent process. And later, it will be lifted up by the increasing pressure drag. The object has a larger pressure drag in low speed. As the spherical particles in the uniform motion, the liquid force which the particles mainly affected by is the pressure drag.

The pressure drag is defined as:

\[ F = \frac{CA\rho v^2}{2} \]

---

**Figure 3-1**  Schematic view of the moment of a flat stone’s contacting the surface of water

### 3.1.1 The Depth of the Immersed Edge

According to the physical and mechanical principle, during collision process between the stone and water, the pressure drag is
\[ \vec{F} = \frac{C_r A \rho v^2}{2} \tau + \frac{C_a A \rho v^2}{2} n \]  

(1)

When \( \alpha \) is constant, the motion equation in the \( z \) direction is

\[ m \frac{d^2 z}{dt^2} = -mg + F_z = -mg + \frac{1}{2} \rho A (C_n \cos \alpha - C_r \sin \alpha) v^2 \]  

(2)

The regular shape of skipping object is circle or rectangle. In order to linearize Equation (2), we use the shape of rectangle instead of the shape of circle. That means

\[ A = \frac{-dz}{\sin \alpha} (z < 0) \]

Meanwhile, according to Bouquet’s theory, \( \alpha \) and \( \beta \) should be sufficiently small. So

\[ \cos \alpha = 1 \]

\[ v_x = v \cos \beta \approx v \]

Equation (2) is rewritten as

\[ m \frac{d^2 z}{dt^2} = -mg - kz \]  

(3)

We define \( k \) as

\[ k = \frac{d}{2 \sin \alpha} \rho v^2 (C_n \cos \alpha - C_r \sin \alpha) \approx \frac{d}{2 \sin \alpha} \rho v_{n, \alpha}^2 C_n. \]

Equation (3) is the simple harmonic motion equation with external force, rewritten as:

\[ \frac{d^2 z}{dt^2} + \frac{k}{m} (z + \frac{mg}{k}) = 0 \]  

(4)

The general solution of Equation (7) is

\[ z(t) = -\frac{g}{\omega^2} + B_1 \cos \omega t + B_2 \sin \omega t \]  

(5)

Where

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{C_r d \rho v_{n, \alpha}^2}{2m \sin \alpha}} \]  

(6)
Substituting the initial conditions:

\[ t = 0, \quad z = 0, \quad v_z = v_{oc} \]

We define \( \Phi \) as

\[
\cos \phi = \frac{g/\omega^2}{\sqrt{\left(\frac{g}{\omega^2}\right)^2 + \left(\frac{v_{oc}}{\omega}\right)^2}}, \quad \sin \phi = \frac{v_{oc}/\omega}{\sqrt{\left(\frac{g}{\omega^2}\right)^2 + \left(\frac{v_{oc}}{\omega}\right)^2}}
\]

We get

\[
z(t) = \frac{g}{\omega^2} + \sqrt{\left(\frac{g}{\omega^2}\right)^2 + \left(\frac{v_{oc}}{\omega}\right)^2} \cos(\omega t - \phi)
\]

It is easy to show that the maximal depth attained by the particle during the collision is

\[
|z_{max}| = \frac{g}{\omega^2} + \sqrt{\left(\frac{g}{\omega^2}\right)^2 + \left(\frac{v_{oc}}{\omega}\right)^2} = \frac{g}{\omega^2} \left[ 2 + \frac{1}{2} \left(\frac{\omega v_{oc}}{g}\right)^2 \right]
\]

3.1.2 The Minimum Speed of Stone Escaping from the Water Surface

As the stone collides into the water surface, it will suffer from the joint action of the pressure and the gravity. The energy will be reduced as the stone contacting the water surface each time. Therefore, the critical bounce condition can be written as

\[ |z_{max}| < d \sin \alpha \]

(1) Taking account of \( \beta \) small enough, we get

\[ v_{oz} = 0 \]

Substituting \( v_{oz} = 0 \) and \( |z_{max}| = \frac{2g}{\omega^2} < d \sin \alpha \) into Equation (8), we get

\[ v_{at}^2 > \frac{4mg}{pd^2C_n} \]

We get conclusion that the critical bounce condition of the initial horizontal velocity
component is
\[ v_{ox}^2 > \frac{4mg}{pd^2C_n}. \]

(2) If \( \beta \) cannot be ignored and \( v_{oc} \neq 0 \), then
\[
|\tau_{max}| = \frac{g}{\omega^2} \left[ 1 + \left( \frac{\omega v_{oc}}{g} \right)^2 \right] < d \sin \alpha
\]
\[
\tan \beta = \frac{v_{oc}}{v_{ox}}
\]

Substituting the above conditions into Equation (8):
\[
v_{ox} = \sqrt{\frac{4mg}{\rho d^2C_n}} \sqrt{1 - \frac{2m \tan^2 \beta}{\rho d^2C_n \sin \alpha}}
\]

We get the conclusion that the critical bounce condition of the horizontal initial velocity component is
\[
v_{ox} = \sqrt{\frac{4mg}{\rho d^2C_n}} \sqrt{1 - \frac{2m \tan^2 \beta}{\rho d^2C_n \sin \alpha}}
\]

The time of the collision between stone and water surface is
\[
t = \frac{z(t)}{v_{oz}} = \frac{2}{\omega} \left[ \pi - \arcsin \left( \frac{|v_{oc}|}{\sqrt{\left( \frac{g}{w} \right)^2 + (v_{oc})^2}} \right) \right]
\]

When \( v_{oc} \) is sufficiently small,
\[
t \approx \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2m \sin \alpha}{C_n \rho v_{ox}^2 b}}
\]

That is, the collision time is approximately equal to one simple harmonic motion cycle, \( T \approx t \).
In the collision process, the liquid pressure drag acted on the stone in the \( z \) direction is 
\[ F = -kz. \] Within the time \( t \), the average liquid pressure drag is \( \bar{F} = -kz \).

We have 
\[ \cos(\omega t - \phi) \approx \cos(2\pi - \phi) = 0 \]

Therefore,
\[
\begin{align*}
\bar{F}_z &= -k \frac{g}{\omega^2} = k \frac{mg}{k} = mg \\
\end{align*}
\]

### 3.1.3 The Number of Bounces

Experience tells us that the stone will skip continuously if the initial velocity of stone is large enough. However, there is resistance in the water, the speed in the \( x \) direction will decrease with the increasing number of the collision times between the stone and water surface, and the distance between the two contact points of the water surface will gradually decrease.

From Equation (2), the pressure drag between water and stone in the \( x \) direction is
\[
F_x = -\frac{\rho w AC_v v^2 \sin \alpha}{2} - \frac{\rho w AC_v v^2 \cos \alpha}{2}
\]

If \( \beta \) is sufficiently small, then \( v \approx v_{\alpha x}^2 \). We have Equation (2) and the flowing equations.
\[ F_x \approx -\frac{dz \rho \alpha C \alpha v^2 \alpha}{2 \sin \alpha} \]

\[ F_z = -k_z \approx -\frac{dz \rho \alpha C \alpha v^2 \alpha}{2 \sin \alpha} \]

Then we obtain

\[ F_x \approx \frac{C_x}{C_n} F_z \]

The average force in the \( x \) direction can be calculated as:

\[ \bar{F}_x \approx \frac{C_x}{C_n} \bar{F}_z \approx \frac{C_x}{C_n} mg \]

Since the collision time is extremely short and the velocity is unrelated to the effect of the angle of stone, the adjacent collision in the \( z \) direction is elastic collision, \( v_{\alpha z} \) is unchanged. That is, the velocity in the \( x \) direction is kept constant. The horizontal stone motion distance during collision process is equal. The power of liquid resistance obtained in collision process can be written as:

\[ W = -\bar{F}_x S = -\frac{C_x}{C_n} mg v_{\alpha z} t \]

From the kinetic energy theorem, the description of the initial to the \( n \)th bounce process is expressed as

\[ \frac{1}{2} m v_{\alpha x}^2 - \frac{1}{2} m v_{\alpha z}^2 = -\frac{C_x}{C_n} m g n l \]  

(13)

And

\[ l \approx v_{\alpha x} t = 2\pi \sqrt{\frac{2 m \sin \alpha}{C_n \rho d}} \]  

(14)

is the horizontal stone motion distance during the collision process. It can be seen that \( l \) is constant regardless of the size \( n \) when \( \alpha \) is fixed. From Equation (12), the maximum number of
bounces of stone can be obtained as

\[ n = \frac{C_n v_{ox}^2}{2C_r gl} = \frac{C_n v_{ox}^2}{4\pi C_r g} \sqrt{\frac{C_r \rho d}{2m \sin \alpha}} \]  \hspace{1cm} (15)

**3.1.4 The Distance Between Two Adjacent Bounces in the Horizontal Direction**

The horizontal velocity \( v_{ox} \) is constant and the velocity perpendicular to the water surface \( v_{oz} \) go with parabolic movement after the nth collision, this process is only affected by the gravity.

From the physical kinematic formulas

\[ s_n(t) = v_{ox} t_n, \quad z_n(t) = v_{oz} t_n - \frac{1}{2} g t_n^2 (z > 0) \]

We obtain

\[ t_{n+1} = \frac{2|v_{oc}|}{g}, \quad s_n = v_{oz} t_{n+1} = \frac{2v_{oz}|v_{oc}|}{g} \]

From the Equation (12), we get

\[
\begin{aligned}
    v_{nx} & = v_{ox} \sqrt{1 - \frac{2C_n n g l}{C_n v_{ox}^2}} = v_{ox} \sqrt{1 - \frac{n}{n_{max}}} \\
    s_n & = v_{nx} t_{n+1} = \frac{2v_{ox}|v_{oz}|}{g}
\end{aligned}
\]  \hspace{1cm} (16)

We get conclusion that the horizontal distance between two adjacent bounces of the stone will decreases with the increasing bounce number with fixed initial velocity. The ripples become intensive with the increasing bounce number.
4 Experimental Results and Analysis

4.1 Experimental Materials

The home-made skipping robot was shown in Figure 4-1. The body frame of it was made of aluminum alloy sheet. The different slant angles can be adjusted. The emitting device is composed of two motors which will lead transmission gears to drive the wheels rotating. The home-made skipping object was driven by wheels, and will be transmitted out through the launch trajectory. The program (see Appendix I) was recorded into main controller. Both the emission velocity and angular velocity can be varied by adjusting the speed of rotation in left and right wheels which are controlled by the LED screen controller.

![The home-made skipping robot](image)

**Figure 4-1** The home-made skipping robot

The home-made skipping object. In reality, most of skipping stones are made of thin stone. Taking into account that the variables should be controlled and the skipping object should be easy to emission, the plastic bottle caps with modification and uniform material are used to act
as the skipping objects in this study. In this way, the mass and the roughness of contact surface can be varied. Furthermore, as shown in Figure 4-2, skipping objects of different shapes were printed by 3D printer in order to explore the influence of different shapes.

![Figure 4-2 Self-made skipping objects](image)

The transparent acrylic sink. In order to capture the whole movement of stone in different angles, a long enough and transparent sink is necessary in the experiment. The experiment phenomena can be observed better as the sink is long enough. With the restrictions of the cost, materials and place of production, an acrylic sink with the size of 240cm*100cm*25cm is made.
in this experiment.

![The transparent acrylic sink](image)

Figure 4-3  The transparent acrylic sink

PCO. 1200s high speed camera of Germany PCO Company. In this experiment, we need to capture the high-speed moving process of skipping. We borrowed PCO.1200s high speed camera of the Germany PCO company from College of mechanical engineering and automation, Fuzhou University. The camera was composed of a cam, external intelligent power, involving image interface software Camware. The Camware was used for controlling the camera to take pictures and adjusting the frame rate (1 frames/s ~ 20000 frames/s). After debugging, the frame rate was set to 500 frames/s in this experiment. Due to the exposure time is extremely short in the high-speed camera, halogen lamps were added as supplemental light source.

![PCO.1200s high-speed camera](image)

Figure 4-4  PCO.1200s high-speed camera
4.2 Experimental Procedure

(1) The sink will be filled with the water to the 2/3 depth of the sink, the light and background cloth were arranged, the high-speed camera was set up at the same height of water surface, the software parameters had been debugged.

(2) The skipping robot should be placed on the launch pad steady, and connected to the regulated power supply whose voltage was set up at 8V. The rotate speeds of two wheels were adjusted through the LED screen controller to achieve the controlling of velocity and angular velocity at the moment of the skipping object emission. The slant angle of the skipping object can be controlled through adjusting the angle supports.

(3) By adjusting the position of the high-speed camera, the dynamic process of the stone skipping water can be captured.
4.3 Experimental Date Analysis and Discussion

4.3.1 Shape Influence

In the lived experience, many people will choose the slightly curved tiles in stone skipping. It seems that such shape of stone would get a better performance. In order to verify this result, we use the 3D printer to print the different shapes of stone, as shown in figure 4-5.

![Different shapes of stones](image)

We repeated this experiment at the same conditions, the following results were obtained as Table 4-1.

<table>
<thead>
<tr>
<th>Type</th>
<th>n (bounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plat</td>
<td>5.28 ± 1.32</td>
</tr>
<tr>
<td>Round</td>
<td>1.77 ± 0.23</td>
</tr>
<tr>
<td>Dome</td>
<td>1.85 ± 0.39</td>
</tr>
</tbody>
</table>

Table 4-1

The experimental results show that the flat stone obtained the best performance. In order to make the experiment more convincing, we turned over the round or dome stone to make the flat surface face down and launched them at the same conditions. We find that the number of
bounces of them increase and are similar to that of the flat one.

We think it is mainly the result of different stress surfaces of stones when they hits the surface of water. The stress surface of stones of different shape is shown in Figure 4-6. Schematic diagram of the stress surface between stone and water surface is shown in Figure 4-7.

![Stress surface](image)

**Figure 4-6** The stress surface between stone and water surface

![Stress surface](image)

**Figure 4-7** Schematic diagram of the stress surface

According to Equation (1), the pressure drag in the $z$ direction is

$$F_z = \frac{1}{2} \rho A (C_n \cos \alpha - C_r \sin \alpha) v^2$$

Considering the sufficiently small $\alpha$, we get
\[ F_z = \frac{1}{2} \rho A C_n \cos \alpha v^2 = \frac{1}{2} \rho A_x C_n v^2 \]

It shows that the pressure drag in the \( z \) direction is in proportion to the area of the stress surface in the \( x \) direction.

We can see from Figure 4-6, for the flat stone, the contact surface in the \( x \) direction are larger than the dome stone and the round ones. It results in a larger pressure drag in the \( z \) direction. Therefore, the flat stone has a better performance than the dome or round ones.

Therefore, we used the flat stone in the further experiments.
4.3.2 The Velocity Loss

Figure 4-8 Flying and collision trajectory of a stone

The stone velocity will change when the water surface exerts a force on the stone during collision. Adopting the Image-Pro Plus 6.0 software, we can track the flight trajectory automatically as shown in 4-10. The image x-t and y-t were generated from the flight trajectory as shown in Figure 4-11, Figure 4-12 and Figure 4-13. The abscissa represents the frame numbers (1frame/2ms), the ordinate represents the pixel of image (we need to use the ruler to convert pixel into the displacements).

Figure 4-9 The x-t image of smooth surface stone during the collision process
Figure 4-10 The x-t image of rough surface stone during the collision process

The Figure 4-9 and 4-10 show the x-t images of smooth and rough surface stone during the collision process respectively. It can be seen that both of smooth and rough surface stone move at constant speed in a straight line before or after collision. But the velocity changes of smooth surface stone is smaller than rough during collision process. We use the after-collision velocity to divide the before-collision velocity, the change rate of velocity shown in Table 4-2. The results show that the roughness of contact surface has a certain influence on the stone force in tangential direction. For smooth surface stone, the velocity changes in the horizontal direction are relatively smaller. The smooth surface is an important condition for stone bouncing continuously.

<table>
<thead>
<tr>
<th>Type</th>
<th>$v_{after}/v_{before}$</th>
</tr>
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<tbody>
<tr>
<td>Smooth</td>
<td>0.76±0.05</td>
</tr>
<tr>
<td>Rough</td>
<td>0.56±0.06</td>
</tr>
</tbody>
</table>

Table 4-2 The horizontal velocity changes of different contact surfaces
The velocity change in the vertical direction was shown in Figure 4-11 before and after a short period of collision. The vertical movement can be seen as two approximate uniform linear motions. We get that the ratio range of after-collision velocity to the before-collision velocity is $0.44 \pm 0.15$ by statistics method. The result shows that there is no significant difference between a rough stone and a smooth one. More importantly, the velocity loss in the vertical direction is larger when we compare it with the horizontal one. It means that some unknown factor affects the velocity change. We will discuss it in the following part.
4.3.3 The Change of the Slant Angle

Figure 4-12 Flight behavior at the different angular velocity of stone.

(The red line is the axis of rotation.)

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$t$</th>
<th>0ms</th>
<th>2ms</th>
<th>4ms</th>
<th>6ms</th>
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<td>5rot/s</td>
<td></td>
<td>44.0</td>
<td>48.7</td>
<td>56.4</td>
<td>59.7</td>
</tr>
<tr>
<td>15rot/s</td>
<td></td>
<td>35.7</td>
<td>37.3</td>
<td>37.2</td>
<td>36.8</td>
</tr>
</tbody>
</table>

Table 4-3 The angle of stone from 0ms to 6ms (the unit: °)

In the process of stone skipping, the rotation of stone is an important factor in flying behavior and collision with water.

From the Figure 4-12 and Table 4-3, as the angular velocity is relatively small, the flight behavior of stone is not stable. After increasing the angular velocity, the stable degree of stone flight behavior has been significantly improved. According to the aerodynamics, the rotation can avoid the disturbance from the air turbulence, then the stone can fly smoothly in the original track. In this way, the incident angle can be remained relatively constant.
The stone behavior changes in 0~4ms after colliding into the water. To consider the rotation, the rotation inertia will change the moment when the collision occurred between the stone and water, therefore, the angle $a$ will change. In order to maintain the best movement state of stone, the change of angle $a$ must be as small as possible.

The stone behaviors changes in 0~4ms after colliding into the water at $\omega \approx 5\text{rot/s}$ and $\omega \approx 15\text{rot/s}$ are shown in Figure 4-13 and Table 4-4. It is obvious that the change of angle $a$ is almost 20° at $\omega \approx 5\text{rot/s}$ in 4ms after collision and the stable degree of flight behavior is poor. In contrast, the angle $a$ is almost unchanged at $\omega \approx 15\text{rot/s}$ in 4ms after collision.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$t$</th>
<th>0ms</th>
<th>2ms</th>
<th>4ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 rot/s</td>
<td></td>
<td>6.7</td>
<td>16.2</td>
<td>26.5</td>
</tr>
<tr>
<td>15 rot/s</td>
<td></td>
<td>18.5</td>
<td>19.2</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Table 4-4  The angle of stone from 0ms to 4ms (the unit: °)
In this regard, we conducted the following analysis. Supposing that the stone radius is \( r \), the angular velocity is \( \omega \) which rotating around the central axis, and the horizontal axis is \( l \) which through the center of stone, as shown in the Figure 4-14. Without considering the influence of angular velocity on force sense, the impulse moment \( \Delta L \) in \( l \) direction will remain constant.

\[
\Delta L = \Delta \omega \times L
\]

![Schematic diagram of angular momentum](image)

Figure 4-14  Schematic diagram of angular momentum

The impulse moment can be calculated as:

\[
\Delta L = L \times \Delta \alpha
\]

So we can obtain

\[
\alpha = \frac{\Delta L}{L} = \frac{2\Delta L}{m\omega^2 r^2} \times \frac{1}{\omega^2}
\]

The results showed that the larger angular velocity, the more stable posture of stone and the smaller changes in angle \( \alpha \). In our experiments, the maximum angular velocity \( \omega \) which the experimental device can provide equivalent to about 15 rot/s.
Figure 4-15 The stone behavior changes in 0~16ms after colliding into the water as

\[ \omega \approx 15 \text{rot/s} \]

Figure 4-16 The angle of stone from 0ms to 16ms

Figure 4-15 shows the stone behavior changes in 0~16ms after colliding into the water.

Figure 4-16 is the data of it. The horizontal axis represents the time with the unit of millisecond. The vertical axis represents the slant angle of stone with the unit of degree.
The curve has a shape of ‘U’ in 0-10ms and the slant angle increases in 10-16ms. This is quite confusing. We made a hypothesis that, when the stone is moving on the water, it change the shape of water and turn the water into a temporary track. The stone moving on the water act similarly to a ball moving on the track, or rather a car, as the car changes not only its position but also the slant angle of it.

Figure 4-17 The temporary track of water

We imagine the track of water as Figure 4-17, and the stone is moving on it. In 0~4ms after the stone collides into the water, the slant angle decreases due to a liquid force. Its reaction force exerted on the surface of water forms the track. In 6~16ms the track has formed and the stone moves on the track with its slant angle increases.

Thus, the change of slant angle of stone after colliding into the water is explained.
4.3.3* The ways of measuring the slant angle

In 4.3.3 part, we analyse the slant angle and study the influence of angular velocity on it. It requires us to measure the slant angle. In our early experiment, we laid a protractor against the computer screen to get the data of angle, which is quite inexact, for we couldn’t find where the exact horizontal plane is.

Later we adopted a new measurement. We used a software to capture every frame of the video that the camera shot. Then, just using the MS Paint software, we got the position point on the stone in the form of pixel value. We need to measure two points on the surface of stone, and calculate the slope of it. The slant angle of stone equals to the tangent value of the slope. In this way the measurement became exact.

Now we realize measuring the pixel values equals to measuring the position of cursor in the form of pixel value, and latter provides a potential for us to have the data recorded by computer itself. Using the Quick Macro software, we wrote a program to capture the position of cursor in one click and automatically calculate the slant angle. The program for Quick Macro is shown in Appendix III. The program greatly increase our speed to measure the slant angle of the stone.
4.3.4 The Critical Condition of Stone Skipping

Many people have such experience that stone thrown out will fall into the water directly and without jumping. In this regard, we do the experiment to find some critical condition which make the stone skip the water. Hundreds of experimental data was collected and the result was shown in Figure 4-18 and Figure 4-19.

![Figure 4-18 Domain of the skipping stone in the $a$ and $U_{\text{min}}$ plane](image)

with $m=5.6\,\text{g}$, $d=4\,\text{cm}$, $\omega=15\,\text{rot/s}$

Figure 4-18 shows that the relation between the minimum velocity $U_{\text{min}}$ and $a$ likes U-shape. As $a=20^\circ$, $U_{\text{min}}=2\,\text{m/s}$, the stone can jump out of water. We substitute $m=5.6\,\text{g}$, $d=4\,\text{cm}$, $a=20^\circ$, $\beta=20^\circ$ into Equation (9), then we can get $U_{\text{min}}=0.40\,\text{m/s}$. Taking into account that the other loss factors are not included in this model, it is reasonable that the experimental value is larger than the theoretical one, and the part of the error will be corrected in the following papers. Considering that the other loss factors are not included in this model, if the setting of $a$ is too larger or small, we need the greater emission velocity to make the stone bounce. Stone is difficult to bounce at too small $a$, which is different from the result of previous sensitivity analysis. The reason is that, in sensitivity analysis, we regarded the coefficient $C$ as constant in the pressure
drag calculation. However, these experiments show that the coefficient $C$ may be related to the contact angle.

Figure 4-19 Domain of the skipping stone in the $\alpha$ and $\beta$ plane

with $m=5.6g$, $d=4cm$, $\omega=15rot/s$.

The Figure 4-19 shows the boundary condition in $\alpha$ and $\beta$ in the domain of the skipping stone. The results show that $\beta$ mainly ranges from $20^\circ$ to $30^\circ$, and a few stones can still bounce at $\beta=45^\circ$. The angle value $\alpha=20^\circ$ is coincide with which shown on Figure 4-14. In order to describe the critical condition of stone skipping better, a curve is used to enclose this data point. Only for $\alpha$ and $\beta$ inside the curve can the stone bounce.
4.3.5 The Number of Bounces

For the stone skipping game, people are most concerned about the number of bounces which is also tested in the experiments. Since the length of sink is only 2.4m and it could not satisfy the experimental test obviously, we conduct this verification experiment outdoors.

Form the Equation (14), we get the number of bounces of stone skipping in ideal conditions, and analyze the influence of the mass of stone and the initial horizontal velocity on it respectively.

(1) The influence of the initial horizontal velocity on the number of bounces

Defining \( u_1 \) as

\[
 u_1 = \frac{C_3/2}{4\pi C_l g} \sqrt{\frac{\rho d}{2m \sin \alpha}}
\]

We can get a quadratic function as follows

\[
 n = u_1 v_{ox}^2
\]

(2) The influence of the mass \( m \) on the number of bounces

Defining \( u_2 \) as

\[
 u_2 = \frac{C_3/2}{4\pi C_l g} \sqrt{\frac{\rho d v_{ox}^4}{2 \sin \alpha}}
\]

We can get a functional relationship as follow

\[
 n = u_2 \sqrt{\frac{1}{m}}
\]

This analysis results are based on the ideal conditions, but the actual experiment may be affected by many uncertain factors, we need to measure each set of experiment data several times and record the largest number of bounces as \( n \). The experimental results and theoretical calculation are shown in Figure 4-20 and Figure 4-21.
The relationship between the number of bounces and the initial horizontal velocity with $m = 8\, g$, $d = 4\, cm$, $\alpha \approx 20^\circ$.

The relationship between the number of bounces and the mass with $d = 4\, cm$, $\alpha \approx 20^\circ$, $v \approx 4\, m/s$. 
From the figures, we see that the experimental values are smaller than the theoretical calculation mainly because the theoretical derivation is too idealistic. The air resistance and the action of water have been simplified. The actual forces of stone is more complicated and more energy loss in actual process, which results the experiment number of bounces $n$ is smaller than the theoretical one. Although the results of experiment are overall small, the trend is identical to theoretical results, which shows that the model construction is reasonable.

### 4.3.6 The Flight Path

Figure 4-22 shows top-down view of a stone. From the figure, we could find that the falling points present certain regularity. The distance between two adjacent falling points gets smaller in the latter part of the flight until the stone falls into the water. This may be caused by the loss of the Mechanical energy.

Apart from this, we also discovered from Figure 4-22 that the former part of the stone is close to linear motion, while the latter presents very obvious deviation. It cannot be explained by the simplified model. But later, we will explain it in our correction model.
4.3.7 The Flying Behavior

Figure 4-23 Change of $\alpha$ when the stone closed to the surface of water

(The time between two adjacent figures is 2ms.)
During the process of stone skipping, we noticed a strange phenomenon. A large number of experiments show that when the stone is close to the water surface, it takes a very clear pose in the air, making the angle of hydroplaning and surface of water mostly vary from 5° to 30°.

For the convenience of description, the clockwise angle from the direction of the horizontal velocity to the horizontal surface is positive. Figure 4-23 show different situations of water entry. In the first one, the angle of hydroplaning and surface of water is about 0°, and the angle is adjusted to 20° when it enters the water. In the second one, the angle of hydroplaning and surface of water is about -20°, and it is adjusted to 0° when it enters the water. Generally speaking, when the speed is quick near the speed of water surface, the adjusted angle is within 25. When the angle of hydroplaning and water surface is about 0°, and is adjusted to 20° when it enters the water. Moreover, if the angle is between 5° and 30°, the adjusted posture when it enters water is unobvious.

Some data of changing of slant angle is shown in Figure 4-24.
In Figure 4-24, the orange points represent the experiment value, the green line represents the fitting curve by trigonometric function and the blue line represents the smooth curve through all the experiment value. The horizontal and vertical axis represent time and slant angle of stone with a unit of millisecond and degree respectively.

We think that the reason why we need the adjustment of position is because the ground effect can’t be ignored when we considered the light mass of the skipping stone and its quick speed. It is generally agreed that the force-exerting area of the ground effect roughly equals to
its wingspan for a plane. We assume that such is the case for a stone. That means the force-exerting area of the ground effect for the stone roughly equals to the diameter. In our experiment, the diameter of stone skipping is 4cm, and several times experiments show the obvious adjustment of posture usually happens when the stone is 3-5cm away from the water surface, which is about the area of the ground effect. In addition, the change of attack angle is also similar tog when the attack angle is negative, and the force from the side close to the ground raises it; When it is positive from 5° to 30°, the force is relatively steady.

The quantitative analysis of the ground effect is complicated and costs much expense, like simulated experiments in which we replace the original fixed ground with mobile ground, inhaled ground or blowing ground. When people study the ground effect, the method of CFD (Computational Fluid Dynamics) is often adopted. It mostly uses the discrete vortex method to simulate the ground and uses finite difference method or finite volume method to solve Euler Equations and N—S Equations. Limited by our competence and time, here we propose a possible explanation and later we'll have further quantitative analysis.
5 Error Correction

5.1 Analysis of Error Factors

The difference is found by comparing experiments result with theoretical result from the simplified model, but the tendency of experimental value fits with theoretical value. Which shows that the consideration of taking the liquid pressure drag as the main acting force is correct. The error came from the possible influent factors we neglected. Then, the model is partially corrected based on the initial theory.

5.1.1 The Error Factor Causing by the Load of Water-Entry

The material shows that large impact take place at the moment the object hits water surface. The effect of impacting is more than the liquid pressure drag. The water-entry load causes the impact. Based on the mathematical model proposed by Komhauser M., when velocity of cylinder is below 100m/s, the loss of velocity is

\[
\frac{\Delta v}{v} = 1 - e^{-\frac{r}{4\pi}}
\]

(17)

Where

\[
r = \frac{m}{\frac{4}{3} \pi R^3 \rho_w}
\]

From Equation (17), the water-entry load cannot be neglected for the stone which weighs only a few grams, we will consider the factor of the water-entry load in the error correction.

5.1.2 The Error Factor Causing by Different Shape

The shape of stone is taken as square in the previous simplified model calculation in the derivation of similar simple harmonic motion equation from dynamical equation of stone, but
the stone we used in the experiment is circular. Neglecting the effect of the acting of the edge shape of stone on the water, the error caused by the shape mainly lies on the thrust during the stone is immersed in water. In order to simplify the calculation, we introduce a physical quantity $\Psi$, so the error can lies on the difference of the immersed area $A$ integration with $x$ and $x = \frac{z}{\sin \alpha}$. The difference can be obtained by changing the relation of $d$ and $R$ ($d = R$ before correction). We suppose the motion state is the same for the rectangular and circular stone after transforming $d$.

(1) For the square, the immersed area is

$$A_{sq} = \frac{z}{\sin \alpha}$$

We get

$$\Psi_{sq} = \int_{A_{sq}} dx = 32cm^3$$

(2) For the circular, the immersed area is

$$A_{cir} = \frac{1}{2} R^2 \left[ 2 \cos^{-1} \left(1 - \frac{x}{R}\right) \right] - \frac{1}{2} 2 (R - x) \sqrt{R - (R - x)^2}$$

We get

$$\Psi_{cir} = \int_{A_{cir}} dx = 25.13cm^3$$

After correction, $d_{cor}$ is satisfied with $\Psi_{cir} = \Psi_{sq} = \frac{1}{2} d_{cor}^3$, we obtain $d_{cor} \approx 3.7cm$.

Supposing the variation $\delta = \frac{\Delta d}{2R} \approx -7.5\%$, the influences on the following parameters are

$$\frac{\Delta v_{ox}}{v_{ox}} \approx -7.3\%$$  for the critical initial horizontal velocity;

$$\frac{\Delta n}{n} \approx 10.6\%$$  for the number of bounces;

$$\frac{\Delta s}{s} \approx 0.01\%$$  for the horizontal distance of a single bounce.
5.1.3 Effect of Viscous Drag

The liquid resistances to moving objects mainly includes viscous drag, pressure drag and wave drag. In addition to the differential pressure resistance that has been considered in this paper, the effect of wave resistance is almost negligible. In order to find out whether the effect of viscous drag causes a significant impact on the experimental result, we further analyze the problem. The Stokes Equation is approximately uses here, that is

$$F = 6\pi \eta v R$$

(18)

The relative change of water drift velocity is

$$\delta = \frac{\Delta v}{v} = \frac{F \Delta t}{m v} = \frac{6\pi \eta R \Delta t}{m} = 7.54 \times 10^{-4}$$

(19)

So the effect of viscous drag on water drift is negligible.
5.2 Error correction

5.2.1 The Correction of the Number of Bounces

The number of bounces \( n \) derived from Equation (15) varied as shown in Figure 4-16. Comparing the experimental result with theoretical result, the trends of both are the same, the value is smaller totally.

![Schematic view of the collision process of a flat stone contacting the surface of water](image)

Figure 5-1 Schematic view of the collision process of a flat stone contacting the surface of water

Considering the influence of water collision load on \( v_{ox} \), the ratio of the velocity of \( x \) direction to the velocity of \( z \) direction of stone is 2:1 from the experimental analysis. Assuming \( v_x = 2v, v_z = v \), then, the normal direction velocity is

\[
v_\perp = 2v \sin \alpha + v \cos \alpha = 1.62v
\]

So the impulse is

\[
I = m\Delta v_\perp = mv_\perp (1 - e^{-\frac{1}{4t}}) = 1.62mv(1 - e^{-\frac{1}{4t}})
\]

Without the neglecting of perpendicular velocity, from the impulse vector graph, Figure 5-1, we have

\[
I_z = I \sin \alpha
\]

The variation of \( v_{ox} \) is
\[ \delta = \frac{\Delta v_x}{v_x} = 0.81(1 - e^{-\frac{1}{4r}}) \sin \alpha \]  

(20)

Then the correction velocity is

\[ v_{x,\text{cor}} = v_x - \Delta v_x = v_x[1 - 0.81(1 - e^{-\frac{1}{4r}}) \sin \alpha] \]

Substitute above formula into Equation (15), we obtain

\[ n = \frac{c_n}{4\pi g c_v} \sqrt{\frac{c_n \rho d}{2m \sin \alpha}} v_{x,\text{cor}}^2 \left[ 1 - 0.81 \left( 1 - e^{-\frac{1}{4r}} \right) \sin \alpha \right]^2 \]  

(21)

Using the values \( m = 8 \times 10^{-3} \text{ kg} \), \( \alpha = 20^\circ \), we get

\[ v_{\text{cor}} = 0.82 v_{x,\text{cor}} \]

Considering the correction of \( d \), \( \sqrt{d'} = 0.962 \sqrt{d} \), we obtain

\[ n_{\text{cor}} \propto v_{\text{cor}}^2 \sqrt{d} = 0.647 n_{\text{the}} \]

Figure 5-2 The relationship between the number of bounces and the initial horizontal velocity with \( m = 8g \), \( d = 4\text{ cm} \), \( \alpha \approx 20^\circ \) after correction

The relationship between the number of bounces and the initial horizontal velocity after
correction is shown on Figure 5-2.

The relationship between the number of bounces and the mass after correction is shown on Figure 5-3.

![Figure 5-3](image)

Figure 5-3 The relationship between the number of bounces and the mass

with $d = 4\text{cm}$, $\alpha \approx 20^\circ$, $v \approx 4\text{m/s}$ after correction

As we can see from the above two figures, the results after correction is fit well with the experimental results.

### 5.2.2 Correction on Critical Velocity $v_{oz}$

Due to $|z_{\text{max}}| = d \sin \alpha$, and $d = 0.04m$, $\alpha = 20^\circ$, we get the theoretical value $v_{oz(\text{ori})} \approx 0.52\text{m/s}$.

But in this case, the experimental value of vertical velocity is $v_{oz(exp)} \approx 1.5\text{m/s}$.

Comparing theoretical value with experimental values, we find that the experimental error is large, so we consider the water impact and have $m = 8 \times 10^{-3}\text{kg}$, $R = 2 \times 10^{-2}\text{m}$, $\rho =$
Then,

\[ \delta = \frac{\Delta v}{v_{zc}} = 0.649 = 64.9\% \]

We can see that the vertical speed of the stone reduces 64.9\% at the moment of contacting with water. Therefore, after the impact into the water,

\[ v_{oz\text{(cor)}} = (1 - \alpha_z)v_{oz\text{(ori)}} = 0.53 \text{m/s} \]

Considering accidental error, we can hold \( v_{oz\text{cor}} = v_{oz\text{ori}} \).

**5.2.3 Correction on Minimum Critical Velocity \( v_{ox} \)**

According to simplified model, when \( \alpha = 20^\circ \), theoretical value of critical velocity in the horizontal direction is \( v_{the} \). Considering the correction on \( d \), the velocity becomes \( v_{cor} = 1.075v_{the} \).

If we take the load of water entry into further consideration, the corrected velocity is

\[ U_{cor(ox)} = \frac{U_{the(ox)}}{1 - \delta}, \text{ then the variation of velocity:} \]

\[ \delta = \frac{\Delta v}{v} = 1 - e^{-\frac{1}{4r}} \approx 0.776 \]

Then

\[ U_{cor(ox)} = 4.46U_{the(ox)} \]

In this situation, due to the theoretical value \( v_{the(ox)} = 0.38 \text{m/s} \), and consider the area correction, we get

\[ U_{cor(ox)} = 1.82 \text{m/s} \]

So

\[ U_{cor} = 1.92 \text{m/s} \]
And the relative error is

\[ \eta = \frac{U_{\text{cor}} - U_{\text{exp}}}{U_{\text{exp}}} = -5\% \]

Which is small and acceptable.

Considering different \( \alpha \), we modify the results of Figure 4-14. From Equation (20), theoretical value of critical velocity is revised to

\[ U = \sqrt{\frac{4mg/\rho d^2 C_n}{1 - \frac{2m\tan^2 \beta}{\rho d^2 C_n \sin \alpha}} \left(1 - \delta\right) \cos \alpha \cos \beta} \]

The graph of critical velocity is shown on Figure 5-4.
5.2.4 Correction on the Deflected Trajectory

5.2.4.1 Analysis

According to Figure 4-18, the stone bounced five times while the trajectory deflected $d_{\text{exp}} = 17.7 \, \text{cm}$. We have tried to use the viscous drag to calculate it, but the force is so small that it cannot make the stone deflected such an obvious distance. Therefore, we would try to find other possible effects.

From the discussion above, we find that the stone will be affected by the load of water entry. The nutation of stone is caused by a moment of force during the process of loading. It will make the stone rotated at a little angle (Figure 5-5), so there will be a component of pressure drag in the direction which is parallel to the launch direction. The track deflection of the stone may be caused by this component. And then, the fluid drag will stop the nutation from keeping going on. So the effect of nutation is neither too much nor too small so as to be ignored.

5.2.4.2 Model Building

For the nutation problem is too complex to be quantitatively calculated, so we used the
Realflow software to simulate it.

Figure 5.6  The scene of the simulation

Figure 5.6 shows the scene of the simulation. We used a moving particle to simulate the water—the particle would crash to the border of the roundness, which just like the water entry process.

Figure 5.7  The control module

To make it more similar to the fact, we used this module (Figure 5.7) to set a variety of physical parameters of the roundness and the particle. Later, the observing work will be done here.
Firstly, we set the roundness with a mass of 8 grams, a slant angle of 30 degrees, a radius of 2 centimeters and an angular velocity of 15 rots per second. The particle was set with a mass of 1 kilogram and a speed of 5 centimeters per second.

Secondly, the impulse of stone caused by the load of water-entry was

\[ I = m\Delta v = m \times 0.649v_\perp = 7.8 \times 10^{-3}\text{N} \cdot \text{s} \]

5.2.4.3 The Observing Work

![Rotation Module](image)

Figure 5-8  The rotation module

We need to observe the dip angle to get the nutation situation, but we could only get the data which showed the components of the angle (Figure 5-8). To get the dip angle, we need to do some geometric calculation and a program was written for it in Visual Basic language, which is shown in Appendix IV.

We only need to put the components of the angle (the first and third grid in Figure 5-8) into ‘Text1’ and ‘Text2’, and click the ‘Command1’. The value of the dip angle will show in ‘Label1’.

5.2.4.4 The simulation data

Using the tools above, we get this graph (Figure 5-10), which shows that the angle changing along with time. The balance point is \( \alpha_0 \approx 23^\circ \), and the amplitude is \( A \approx 3.2^\circ \).
5.2.4.5 Calculation

In Figure 5-11:

The plane ABC stands for the plane of the stone

The plane BCD stands for the horizontal plane

The line EF stands for the radius which goes through the contact point

G is the projection of F.

(1) The $\omega$

Before the water-entry happen,

$$\omega_0 = 15r/s = 5400°/s,$$

Because of the conservation of angular momentum (vertical of the roundness),

$$I_t\omega_t = I_0\omega_0$$

For

$$I_t = I_0 \cos 23°$$

We get

$$\omega_t = \frac{\omega_0}{\cos 23°} \approx 5866.34°/s$$

(2) The $\theta$

From the video we can know that time only continues about four microseconds from the water-entry to the ‘water slide’ begin to stop the nutation. So

$$\theta = \omega_t \times \Delta t \approx 23.47°$$
(3) The Impulse

The impulse caused by the pressure drag is

\[ I_1 = m\Delta v = 8.4 \times 10^{-3} \text{kg} \cdot \text{m/s} \]

The component of the impulse, which is in the z direction is

\[ I_z = I_1 \sin \theta \sin \alpha_0 \]

And

\[ \Delta v = \frac{I_z}{m} \approx 0.17 \text{m/s} \]

So

\[ v_z = 0.17 \text{m/s} \]

(4) The Deflection Distance

For the horizontal velocity is

\[ v_h = 5 \text{m/s} \]

And the total distance is

\[ s = 1.7 \text{m} \]

So the total moving time is

\[ t = \frac{s}{v_h} = 0.34 \text{s} \]

In the five-time bounces, the average velocity in z direction is

\[ \bar{v} = \frac{v_z + 5v_z}{2} = 0.51 \text{m/s} \]

The deflection distance is

\[ d_{\text{the}} = \bar{v}t \approx 0.1683 \text{m} = 17.37 \text{cm} \]

And relative error is

\[ \delta = \frac{\Delta d}{d_{\text{exp}}} = \frac{d_{\text{the}} - d_{\text{exp}}}{d_{\text{exp}}} = -2\% \]
6 The Analysis of the World Record Video

According to the simplified model and the correction, we have successfully made the theoretical line coincide with most of the experiment value. So what about a real stone thrown by a real person in a real scene? In the next part we going to analyse the present world record created by Gang YouShi.

6.1 The Analysis of the Initial Horizontal Velocity

In reality, a stone has a diameter of about 7 centimeters, a thickness of 0.8 centimeters and a density of 2.5 grams per cubic centimeter. Then we could get the mass

\[ m = \frac{1}{4} \rho \pi d^2 l = 98 \text{g} \]

We assume the stone was thrown with the best slant angle \( \alpha = 20^\circ \), and according to Equation (21), we get

\[ v_{ox} = \sqrt{\frac{4 \pi g C_t}{C_n} \left(\frac{2m \sin \alpha}{C_n \rho d}\right)^{0.5} \left[1 - 0.81 \left(1 - e^{-\frac{1}{4\pi}} \sin \alpha\right)\right]^{-2} \times n} \]

Substituting the world record \( n = 91 \), we got \( v_{ox} = 20.9 \text{m/s} \).

![Figure 6-1 the screenshots of the world record video](image-url)
Then we measured the screenshots of the world record video. Some main screenshots is shown in Figure 6-1. We assumed he was 1.7 meters in height. By choosing an object of reference and measuring the relative position between the stone the reference object, we got the data which is shown in Table 6-1.

<table>
<thead>
<tr>
<th>$t$ (ms)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{ox}$ (m/s)</td>
<td>25.5</td>
<td>19.8</td>
<td>18.4</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Table 6-1 The data of initial velocity of the world record

Based on simple statistics calculation, we got $v_{ox} = 21.6 \pm 2.7 \text{m/s}$. The theoretical value falls on the experiment range, which shows the correctness of our theory.

6.2 The Instability of the Bounce Distance

According to the Figure 6-2 and Figure 4-22, we can see that sometimes the distance of bounces in the world record are short, but sometimes it can be even longer.

Here is our conjecture:

The condition that cavitation can be produced is

$$\xi \geq \sigma$$

Where
\[ \xi = \left( \frac{V_b}{V_0} \right)^2 - 1 \]

In this formula, \( V_b \) is the stone’s velocity, \( V_0 \) is the freestream velocity.

And

\[ \sigma = \frac{p_0 - p_v}{\frac{1}{2} \rho_w V_0^2} \]

\( p_0 \) is the atmospheric pressure, \( p_v \) is the saturated vapor pressure of water, \( \rho_w \) is the density of water.

So we get

\[ V_b \geq 21m/s \]

For the stone has horizontal velocity, and it is rough enough to drive the water. So it is possible to produce the cavitation.

In some conditions, the cavitation can appear in the head of the stone. For the pressure in the cavitation is the saturated vapor pressure of water, which is much smaller than the atmospheric pressure. So the head of the stone will be pressed down by the pressure difference. This will cause a phenomenon called ‘whip’. Then the bounce distance will de reduce a lot.

Because the separation line of the cavitation is changing irregularly, which can be caused by wave, wind and so on, the influence caused by the pressure difference is also changing irregularly. So the bounce distance has instability.

In our experiment, the cavitation can be produced needs

\[ V_b \approx 14m/s \]

But the initial vertical velocity is only about 5m/s. So the cavitation cannot come into being.

So the bounce distance is in regular degressive trend.
7 Conclusion

In experience, we followed these rules: choose a stone which is flat or circular, and throw them fast and with a finger to give the stone a spin.

The experiment results of stone skipping show that in a way the simplified physical model which we built is consistent with the experimental results. However, the quantitative calculation is still in the rough. It mainly due to stone skipping involves different fluid properties on the surface of the contact, and it involves complicated fluid mechanics. For example, the fluid is easy to flow and it has viscosity and compressibility. When the fluid flows, it has many different states, like laminar and turbulent flow. When a relative motion happens between fluid and objects in the fluid, they will affected by the gravity, resistance, pressure of water. The resistance has viscous resistance, differential pressure resistance and wave resistance. Therefore, if we want to analyze the fluid we need to use CFD (Computational Fluid Dynamics) method to solve some problems of complex hydrodynamics physical model. For this point, as our guide teacher said, we needed to seize the main contradiction, idealized the practical problems, and put ideal results into practical research in order to study and analyze the complicated problems.

This paper is based on our idea, and we use some basic mechanics principles to establish a simplified physical model which simulated the process of stone skipping. We regarded the force from water as the differential pressure in fluid resistance. Simplified the stress analysis and set up experimental device at the same time and compared experimental results with the actual theory. By analyzing on Features of stone skipping, we got the following main conclusions:

(1) Compared with the curved stone, the flat one has a better performance.
(2) The angular velocity plays an important role in the process of stone skipping. It can stabilize the flight behavior.

(3) The roughness of the contact surface has a significant effect on the movement of stone in the horizontal direction. The smooth stone has more bounces.

(4) For stone skipping, the velocity which is under a certain point or the improper angle will lead to a failure in stone skipping. We get that when the angle is 20° and the faster the velocity is, the better stone skipping we will get.

(5) The number of bounces of stone skipping is related to the velocity and the mass. If other conditions remain unchanged, the greater velocity and the smaller mass the stone has, the more bounces we will get.

(6) The slant angle of stone after entering the water decreases and then increases. It can be described by the model of the temporary track of water.

(7) The movement of the stone skipping is also affected by the load of water entry. The theoretical results are more consistent with the experimental ones and the world record by considering the effect of the load of water entry.

(8) The deflected trajectory may be caused by one component of the pressure drag which is parallel to the launch direction, which results in the nutation of stone.

(9) Some unexpected phenomena are explained. For example, the flight behavior of stone can adjust automatically before falling into the water, which may be affected by the ground effect. Another one which is inconsistent with the simplified model is that the distance of bounces in the world record video are sometimes short and sometimes long, and a possible reason is the influence of cavitation.
References


[3].Christophe Clanet, Fabien Hersen , and Lyderic Bocquet, The revealed secrets of stone skipping,


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Appendix I  Self-made Skipping Robot Control Program

```c
#include "Main.h"

void main ( void )
{
    InitLCD ( 1 ) ;
    InitLCD ( 2 ) ;
    StartLCDButtonsWatcher ( 1 ) ;
    StartLCDButtonsWatcher ( 2 ) ;
    SetLCDLight ( 1 , 1 ) ;
    SetLCDLight ( 2 , 1 ) ;
    SetLCDText ( 1 , 1 , " - ON/OFF +" ) ;
    SetLCDText ( 2 , 1 , " - +" ) ;
    SetLCDText ( 1 , 2 , "Battery Unplugged" ) ;
    SetLCDText ( 2 , 2 , "Battery Unplugged" ) ;
    SetLCDText ( 1 , 2 , "Battery Unplugged" ) ;
    while ( battery == 0 )
    {
        battery = GetMainBattery ( ) ;
    }
    SetLCDText ( 1 , 2 , "LEFT - Vot=%f" , battery ) ;
    SetLCDText ( 2 , 2 , "RIGHT - Vot=%f" , battery ) ;
    while ( 1 )
    {
        GetLCDButtonsWatcher ( 1 , \&lcd1_left , \&lcd1_middle , \&lcd1_right ) ;
        if ( lcd1_middle == 1 && per == 1 )
        {
            dipan ( control1 , control2 ) ;
            while ( lcd1_middle == 1 )
            {
                GetLCDButtonsWatcher ( 1 , \&lcd1_left , \&lcd1_middle ,
                \&lcd1_right ) ;
                GetLCDButtonsWatcher ( 2 , \&lcd2_left , \&lcd2_middle ,
                \&lcd2_right ) ;
            }
            per = -per ;
        }
        GetLCDButtonsWatcher ( 1 , \&lcd1_left , \&lcd1_middle , \&lcd1_right ) ;
        if ( lcd1_middle == 1 && per == -1 )
        {
            dipan ( 0 , 0 ) ;
            battery = GetMainBattery ( ) ;
            SetLCDText ( 1 , 2 , "LEFT - Vot=%f" , battery ) ;
            SetLCDText ( 2 , 2 , "RIGHT - Vot=%f" , battery ) ;
        }
}
```

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while ( lcd1_middle == 1 )
{
    GetLCDButtonsWatcher ( 1 , &lcd1_left , &lcd1_middle , &lcd1_right ) ;
    GetLCDButtonsWatcher ( 2 , &lcd2_left , &lcd2_middle , &lcd2_right ) ;
    per = -per ;
}
GetLCDButtonsWatcher ( 1 , &lcd1_left , &lcd1_middle , &lcd1_right ) ;
if ( lcd1_left == 1 && per == -1 )
{
    control1 = control1 - 5 ;
    if ( control1 <= 0 )
    {
        control1 = 0 ;
    }
    while ( lcd1_left == 1 )
    {
        GetLCDButtonsWatcher ( 1 , &lcd1_left , &lcd1_middle , &lcd1_right ) ;
    }
    dipan ( control1, control2 ) ;
}
GetLCDButtonsWatcher ( 2 , &lcd2_left , &lcd2_middle , &lcd2_right ) ;
if ( lcd2_left == 1 && per == -1 )
{
    control2 = control2 - 5 ;
    if ( control2 <= 0 )
    {
        control2 = 0 ;
    }
    while ( lcd2_left == 1 )
    {
        GetLCDButtonsWatcher ( 2 , &lcd2_left , &lcd2_middle , &lcd2_right ) ;
    }
    dipan ( control1, control2 ) ;
}
GetLCDButtonsWatcher ( 1 , &lcd1_left , &lcd1_middle , &lcd1_right ) ;
if ( lcd1_right == 1 && per == -1 )
{
    control1 = control1 + 5 ;
    if ( control1 >= 127 )
{  
    control1 = 127 ;
}
while ( lcd1_right == 1 )
{
    GetLCDButtonsWatcher ( 1 , &lcd1_left , &lcd1_middle ,
    &lcd1_right ) ;
    dipan ( control1 , control2 ) ;
}
GetLCDButtonsWatcher ( 2 , &lcd2_left , &lcd2_middle , &lcd2_right ) ;
if ( lcd2_right == 1 && per == -1 )
{
    control2 = control2 + 5 ;
    if ( control2 >= 127 )
    {
        control2 = 127 ;
    }
    while ( lcd2_right == 1 )
    {
        GetLCDButtonsWatcher ( 2 , &lcd2_left , &lcd2_middle ,
        &lcd2_right ) ;
    }
    dipan ( control1 , control2 ) ;
}
}

#include "Main.h"

void dipan ( int did1 , int did2 )
{
    SetMotor ( 1 , did1 ) ;
    SetMotor ( 10 , -did2 ) ;
    SetLCDText ( 1 , 1 , "- ON/OFF +" ) ;
    SetLCDText ( 2 , 1 , "- +" ) ;
    battery = GetMainBattery ( ) ;
    SetLCDText ( 1 , 2 , "LEFT=%d" , did1/5 ) ;
    SetLCDText ( 2 , 2 , "RIGHT=%d" , did2/5 ) ;
}
Appendix II  MATLAB Program

c2=1/4/pi/9.8*(1000*0.04/2/0.008/sin(20/180*pi))^(1/2);
x2=1.5:1:5.5; y2=c2*x2.^2; plot(x2,y2,'linewidth',2);
xlabel('x direction velocity v (m/s)');
ylabel('n');
hold on;
x=[2 3 4 5];
y=[1 4 7 11];
scatter(x,y);
hold on;
values = spcrv([x(1) x x(end)];[y(1) y y(end)]),2);
plot(values(1,:),values(2,:), 'g');
hold on;
y3= 0.647*y2;
plot(x2,y3, '-r');
legend('theoretical line','2nd order fit','correction data','experimental data')
figure;
c3=1/4/pi/9.8*4*4*(1000*0.04/2/sin(20/180*pi))^(1/2);
x3=1:0.5:15; y3=c3*1./(x3/1000).^0.5;
for i=1:29
    r(i)=x3(i)/1000*3/4/pi/1000/(0.02)^3;
    z(i)=1-exp(-1/4/r(i));
    zz(i)=1-0.81*z(i)*sin(20/180*pi);
    y33(i)=y3(i)*0.962*zz(i)^2;
end
plot(x3,y3,'linewidth',3);
xlabel('m (g)');
ylabel('n');
hold on;
x=[3.1 5.6 8.1];
y=[9 9 7];
scatter(x,y);
plot(x3,y33, '-r');
legend('theoretical line','correction data','experimental data','2nd order fit')

Appendix III  Quick Macro Program

Dim num, out, x1, x2, y1, y2, n, logic
UserVar n = 2 "Times of Measurements in Each Frame"
UserVar logic = DropList{"YES":"1"|"NO":"0"} "If measuring continuous frames"
Call Plugin.Office.OpenXls("D:\OUT.xls")
num = Plugin.Office.ReadXls(1, 1, 1)
frame = Inputbox("Which frame")
Do
    For n = 1 To n
        GetCursorPos x1, y1
        WaitKey
        GetCursorPos x2, y2
        WaitKey
        out = out + Tan((y2 - y1) / (x2 - x1)) / 3.1416 * 180
    Next
    num = num + 1
    out = out / n
    Call Plugin.Office.WriteXls(1, 1, 1, num)
    Call Plugin.Office.WriteXls(1, num + 1, 2, 2 * frame)
    Call Plugin.Office.WriteXls(1, num + 1, 3, out)
    out = 0
    If logic = 1 Then
        frame = frame + 1
    Else
        frame = Inputbox("Which frame")
    End If
Loop
Sub OnScriptExit()
    Call Plugin.Office.CloseXls()
End Sub

Appendix IV Visual Basic Program

Private Sub Command1_Click()
Dim x As Single, y As Single, p As Single, h As Single, z As Single, S As Single, d As Single
x = Text1.Text
y = Text2.Text
a = 1 / Sin(x / 180 * 3.14)
b = 1 / Sin(y / 180 * 3.14)
c = Sqr(a ^ 2 + b ^ 2)
p = 1 / 2 * (a + b + c)
S = Sqr(p * (p - a) * (p - b) * (p - c))
h = 2 * S / c
d = Sqr(h ^ 2 - 1)
z = Atn(1 / d) / 3.14 * 180
Label1.Caption = z
End Sub