Clearly, the principal curvatures are diagonal entries. ∎

Theorem 5.10. *The Gauss curvature of M is given below:*

$$K = -\frac{\varphi'' \psi'^{n-3}}{\varphi^{n-2}} (n \geq 3).$$

Proof. The Gauss curvature is equal to the product of the principal curvatures by definition:

$$K = \prod_{i=1}^{n-1} k_i = -\frac{\varphi''}{\psi'} \cdot (\frac{\psi'}{\varphi})^{n-2} = -\frac{\varphi'' \psi'^{n-3}}{\varphi^{n-2}}.$$

∎

Now, we have derived the expression of the Gauss curvature of M under the φ and ψ parametrization.

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